Tunnelling between non-centrosymmetric superconductors with significant spin-orbit splitting studied theoretically within a two-band treatment

K. Børkje[1] and A. Sudbø[1]

[1]*Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway*

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Tunnelling between non-centrosymmetric superconductors with significant spin-orbit splitting is studied theoretically in a two-band treatment of the problem. We find that the critical Josephson current may be modulated by changing the relative angle between the vectors describing absence of inversion symmetry on each side of the junction. The presence of two gaps also results in multiple steps in the quasiparticle current-voltage characteristics. We argue that both these effects may help to determine the pairing states in materials like $CePt_3Si$, UIr and $Cd_2Re_2O_7$. We propose experimental tests of these ideas, including scanning tunnelling microscopy.

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Superconductors where inversion symmetry is absent and spin-orbit splitting is significant have recently attracted considerable attention.[1,2,3,4,5,6,7,8] Much of this interest was initiated by the discovery of superconductivity in $CePt_3Si$[9] and UIr.[10] In addition to not having an inversion center, band structure calculations on $CePt_3Si$[3] have shown that spin-orbit coupling splits otherwise degenerate bands by 50-200 meV near the Fermi level. This is much larger than k_BT_c, where T_c is the critical temperature for superconductivity. This must be taken into account when describing superconductivity in these systems. These materials also order magnetically, which could influence the nature of the superconducting state. However, at least for $CePt_3Si$, there seems to be little communication between superconductivity and magnetic order.[11]

Another superconductor of interest in this context is $Cd_2Re_2O_7$. This material has a structural phase transition where a center of symmetry is lost.[12] When certain ions in the unit cell are displaced throughout the lattice, internal electric fields are induced, giving rise to spin-orbit splitting of spin-degenerate states. Calculations and photoemission studies[13] have indicated that this splitting will have a significant influence on the electronic band structure. Thus, $Cd_2Re_2O_7$ is similar to the materials mentioned above, although simpler, since it shows no sign of magnetic order.[14] Another pyrochlore superconductor that might fall into this category is KOs_2O_6.[15]

The spin-orbit splitting of otherwise degenerate bands demands a two-band description of superconductivity in these materials. An exotic feature of non-centrosymmetric superconductors with large spin-orbit splitting is the possible absence of a definite parity of the superconducting state.[1,2,6] Experiments have indicated that $CePt_3Si$ might be in such a pairing state, a linear combination of spin-singlet and spin-triplet states, and that the gap may contain line nodes.[8] $Cd_2Re_2O_7$ and KOs_2O_6 seems to be nodeless, however.[14,15]

In the present study, we will investigate tunnelling currents between two superconductors where a two-band description is necessary in both systems. Junctions involving one superconductor with spin-orbit split bands have been studied in Refs. 5 and 16. We will restrict ourselves to intraband Cooper pairing without specifying the microscopic mechanism responsible for this. See Refs. 17 for related work on MgB_2 junctions. Our main focus will be on non-centrosymmetric superconductors with spin-orbit split bands and we will specialise to this case when needed. We find that the critical Josephson current may be modulated by changing the angle between the vectors describing absence of inversion symmetry on each side. This effect is analogous to tunnelling magnetoresistance in ferromagnetic tunnel junctions.[18] We also calculate the quasiparticle current. For temperatures close to zero, the current-voltage diagram may contain several discontinuities determined by the relative size of the two gaps. We claim that both these results may help to determine the possibly novel properties of the superconducting state in materials like $CePt_3Si$, UIr and $Cd_2Re_2O_7$.

The Hamiltonian considered is $H = H_{\rm N} + H_{\rm SC}$, where $H_{\rm SC}$ describes superconductivity. The normal state Hamiltonian is

$$H_{\rm N} = \sum_{\boldsymbol{k}} \phi^\dagger_{\boldsymbol{k}} \left[\varepsilon_{\boldsymbol{k}} + \boldsymbol{B}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}\right] \phi_{\boldsymbol{k}}, \tag{1}$$

where $\phi^\dagger_{\boldsymbol{k}} = (c^\dagger_{\boldsymbol{k}\uparrow}, c^\dagger_{\boldsymbol{k}\downarrow})$, $\varepsilon_{\boldsymbol{k}}$ is the band dispersion, and the vector $\boldsymbol{\sigma}$ consists of the three Pauli matrices. We name the spin quantisation axis the z-axis. The vector $\boldsymbol{B}_{\boldsymbol{k}}$ removes the spin degeneracy from the band $\varepsilon_{\boldsymbol{k}}$. By a transformation to a basis $\tilde{\phi}^\dagger_{\boldsymbol{k}} = (\tilde{c}^\dagger_{+,\boldsymbol{k}}, \tilde{c}^\dagger_{-,\boldsymbol{k}})$ where (1) is diagonal, one finds $H_{\rm N} = \sum_{\lambda=\pm,\boldsymbol{k}} \tilde{\varepsilon}_{\lambda,\boldsymbol{k}} \tilde{c}^\dagger_{\lambda,\boldsymbol{k}} \tilde{c}_{\lambda,\boldsymbol{k}}$. The quasiparticle spectrum is $\tilde{\varepsilon}_{\pm,\boldsymbol{k}} = \varepsilon_{\boldsymbol{k}} \pm |\boldsymbol{B}_{\boldsymbol{k}}|$. We define $B_{\boldsymbol{k},\pm} = B_{\boldsymbol{k},x} \pm {\rm i} B_{\boldsymbol{k},y} = B_{\boldsymbol{k},\perp} {\rm e}^{\pm {\rm i}\varphi_{\boldsymbol{k}}}$.

The vector $\boldsymbol{B}_{\boldsymbol{k}}$ has the property $\boldsymbol{B}_{-\boldsymbol{k}} = -\boldsymbol{B}_{\boldsymbol{k}}$,[4] where $\boldsymbol{B}_{\boldsymbol{k}}$ characterises the absence of inversion symmetry in the crystal. The origin may be that ions are removed from high-symmetry positions, as in ferroelectrics,[12] leading to internal electric fields and thus increased spin-orbit coupling.[19,20] To establish the form of $\boldsymbol{B}_{\boldsymbol{k}}$, point group symmetry considerations may be employed[4] and $\boldsymbol{B}_{\boldsymbol{k}}$ will depend on the direction in which the ions are displaced.

An electron with momentum $\boldsymbol{k}$ will align its spin parallel or antiparallel to $\boldsymbol{B}_{\boldsymbol{k}}$. In a free electron model with the Rashba interaction,[19] the 1D density of states for the $+$ and $-$ band at the Fermi level are equal.[21] Still, we allow these to be unequal, which is the general case.

Let us now turn to the term responsible for superconductivity, $H_{\rm SC}$. We write down the interaction in terms of the long-lived excitations in the normal state

$$H_{\rm SC} = \frac{1}{2}\sum_{\lambda\mu,\boldsymbol{k}\boldsymbol{k}'} V_{\lambda\mu}(\boldsymbol{k},\boldsymbol{k}')\tilde{c}^\dagger_{\lambda,-\boldsymbol{k}}\tilde{c}^\dagger_{\lambda,\boldsymbol{k}}\tilde{c}_{\mu,\boldsymbol{k}'}\tilde{c}_{\mu,-\boldsymbol{k}'}. \tag{2}$$

This model contains only intraband Cooper pairing. Interband Cooper pairs are strongly suppressed if the spin-orbit splitting is much larger than the superconducting gaps, even though the two bands may touch at some isolated points on the Fermi surface.[3] This is the limit we are investigating. Defining $\Delta_{\lambda,\boldsymbol{k}} = -\sum_{\mu,\boldsymbol{k}'} V_{\lambda\mu}(\boldsymbol{k},\boldsymbol{k}')\langle\tilde{c}_{\mu,\boldsymbol{k}'}\tilde{c}_{\mu,-\boldsymbol{k}'}\rangle$, the standard mean field approach gives the total Hamiltonian

$$H = \sum_{\lambda,\boldsymbol{k}}\left[\tilde{\varepsilon}_{\lambda,\boldsymbol{k}}\tilde{c}^\dagger_{\lambda,\boldsymbol{k}}\tilde{c}_{\lambda,\boldsymbol{k}} + \frac{1}{2}\left(\Delta_{\lambda,\boldsymbol{k}}\tilde{c}^\dagger_{\lambda,\boldsymbol{k}}\tilde{c}^\dagger_{\lambda,-\boldsymbol{k}} + \text{h.c.}\right)\right]. \tag{3}$$

Note that $\Delta_{\lambda,-\boldsymbol{k}} = -\Delta_{\lambda,\boldsymbol{k}}$ follows from the fermionic anticommutation relations. In Equation (3), the two bands are decoupled, resulting in Green's functions diagonal in the band indices. This is a result of the mean field approximation. $\Delta_{\pm,\boldsymbol{k}}$ are in general not independent, but related through the self-consistency equations due to the possibility of interband pair scattering.[7]

The relation $\boldsymbol{B}_{-\boldsymbol{k}} = -\boldsymbol{B}_{\boldsymbol{k}}$ ensures that states with opposite momenta within a band have opposite spins. For a spin-1/2 state, the time reversal operator is $\mathcal{K} = -\mathrm{i}\sigma_y\mathcal{K}_0$, where $\mathcal{K}_0$ denotes complex conjugation. Let the original operators transform according to $\mathcal{K} : c^\dagger_{\boldsymbol{k},\sigma} = -\sigma c^\dagger_{-\boldsymbol{k},-\sigma}$ under time reversal. The effect of time reversal on the new operators then becomes $\mathcal{K} : \tilde{c}^\dagger_{\lambda,\boldsymbol{k}} = t_{\lambda,\boldsymbol{k}}\tilde{c}^\dagger_{\lambda,-\boldsymbol{k}}$, where $t_{\lambda,\boldsymbol{k}} = \mathrm{e}^{-\lambda\mathrm{i}\varphi_{\boldsymbol{k}}}$. This means that if $\chi_{\lambda,\boldsymbol{k}}$ is the order parameter for pairs of time reversed states, one finds $\Delta_{\lambda,\boldsymbol{k}} = t_{\lambda,\boldsymbol{k}}\chi_{\lambda,\boldsymbol{k}}$. This gives $\chi_{\lambda,\boldsymbol{k}} = \chi_{\lambda,-\boldsymbol{k}}$. Thus, $\chi_{\lambda,\boldsymbol{k}}$ may be expanded in terms of even basis functions of irreducible representations of the space group.[5]

Define the matrix $\boldsymbol{\Delta}_{\boldsymbol{k}}$ whose elements are the gap functions $\Delta_{\boldsymbol{k},\sigma\sigma'}$ in the original basis where spin is quantised along the z-axis. This may be written as

$$\boldsymbol{\Delta}_{\boldsymbol{k}} = \eta_{\boldsymbol{k},\rm S}\, g + \eta_{\boldsymbol{k},\rm T}(\hat{\boldsymbol{B}}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma})\, g, \tag{4}$$

where $g = -\mathrm{i}\sigma_y$. The first term is symmetric in momentum space and antisymmetric in spin space, whereas the opposite is the case for the last term. Thus, in the absence of spatial inversion symmetry, the order parameters in a spin basis have no definite parity, but is in general a linear combination of singlet (S) and triplet (T) parts.[1,2,6] The singlet and triplet components are determined by $\eta_{\boldsymbol{k},\rm S} = (\chi_{+,\boldsymbol{k}} + \chi_{-,\boldsymbol{k}})/2$ and $\eta_{\boldsymbol{k},\rm T} = (\chi_{+,\boldsymbol{k}} - \chi_{-,\boldsymbol{k}})/2$, respectively. *This means that knowledge of $\chi_{\pm,\boldsymbol{k}}$ and $\boldsymbol{B}_{\boldsymbol{k}}$ could help determine the gap structure and the symmetry of the pairing state.* For non-centrosymmetric materials like $\mathrm{CePt_3Si}$, this is currently a matter of intense study.[3,4,5,6,8]

The normal and anomalous Green's functions for each band are $\mathcal{G}_\lambda(\boldsymbol{k},\omega_n) = -(\mathrm{i}\omega_n + \xi_{\lambda,\boldsymbol{k}})/(\omega_n^2 + \xi^2_{\lambda,\boldsymbol{k}} + |\chi_{\lambda,\boldsymbol{k}}|^2)$ and $\mathcal{F}_\lambda(\boldsymbol{k},\omega_n) = t_{\lambda,\boldsymbol{k}}\chi_{\lambda,\boldsymbol{k}}/(\omega_n^2 + \xi^2_{\lambda,\boldsymbol{k}} + |\chi_{\lambda,\boldsymbol{k}}|^2)$, respectively. These are defined in the standard way, see *e.g.* Ref. 5. ω_n is a fermion Matsubara frequency, $\xi_{\lambda,\boldsymbol{k}} = \tilde{\varepsilon}_{\lambda,\boldsymbol{k}} - \mu$ and μ is the chemical potential.

Consider tunnelling between two such superconductors, A and B. Let system A be described by the Hamiltonian (3). The Hamiltonian of system B is defined equivalently, only with $c_{\boldsymbol{k}\sigma}, \tilde{c}_{\lambda,\boldsymbol{k}} \to d_{\boldsymbol{p}\sigma}, \tilde{d}_{\rho,\boldsymbol{p}}$. Moreover, we allow $\boldsymbol{B}^{\rm A}_{\boldsymbol{k}}$ and $\boldsymbol{B}^{\rm B}_{\boldsymbol{p}}$ to be different. Consequently, even if $\boldsymbol{k} = \boldsymbol{p}$, the spin in a state $+$ or $-$ may be different on sides A and B. The tunnelling Hamiltonian is $H_{\rm T} = \sum_{\lambda\rho,\boldsymbol{k}\boldsymbol{p}}\left(\tilde{T}^{\lambda\rho}_{\boldsymbol{k}\boldsymbol{p}}\tilde{c}^\dagger_{\lambda,\boldsymbol{k}}\tilde{d}_{\rho,\boldsymbol{p}} + \tilde{T}^{\lambda\rho\,*}_{\boldsymbol{k}\boldsymbol{p}}\tilde{d}^\dagger_{\rho,\boldsymbol{p}}\tilde{c}_{\lambda,\boldsymbol{k}}\right)$. The tunnelling matrix elements $\tilde{T}^{\lambda\rho}_{\boldsymbol{k}\boldsymbol{p}}$ depends strongly on the *direction* of $\boldsymbol{k}$ and $\boldsymbol{p}$.[22] Tunnelling is much more probable for a momentum normal to the interface rather than parallel to it.[22,23,24] If we assume that spin is conserved in the tunnelling process, *i.e.* $H_{\rm T} = \sum_{\boldsymbol{k}\boldsymbol{p},\sigma} T_{\boldsymbol{k}\boldsymbol{p}}c^\dagger_{\boldsymbol{k}\sigma}d_{\boldsymbol{p}\sigma} + \text{h.c.}$, we find $|\tilde{T}^{\lambda\rho}_{\boldsymbol{k}\boldsymbol{p}}|^2 = |T_{\boldsymbol{k}\boldsymbol{p}}|^2(1 + \lambda\rho\,\hat{\boldsymbol{B}}^{\rm A}_{\boldsymbol{k}}\cdot\hat{\boldsymbol{B}}^{\rm B}_{\boldsymbol{p}})/2$.

The number operator for band λ in system A is $N^{\rm A}_\lambda = \sum_{\boldsymbol{k}}\tilde{c}^\dagger_{\lambda,\boldsymbol{k}}\tilde{c}_{\lambda,\boldsymbol{k}}$. We define $\dot{N}^{\rm T}_\lambda = \mathrm{i}[H_{\rm T}, N^{\rm A}_\lambda]$, such that the charge current is $I(t) = -e\sum_\lambda\langle\dot{N}^{\rm T}_\lambda\rangle$. To lowest order in the tunnelling matrix elements, standard theory gives $I(t) = I_{\rm qp} + I_{\rm J}(t)$, where $I_{\rm qp} = -2e\sum_\lambda \operatorname{Im}\Phi_\lambda(eV)$ and $I_{\rm J}(t) = 2e\sum_\lambda \operatorname{Im}\left[\Psi_\lambda(eV)\,\mathrm{e}^{2\mathrm{i}eVt}\right]$. The voltage is $eV = \mu_{\rm A} - \mu_{\rm B}$. In the Matsubara formalism, we have

$$\begin{aligned}\Phi_\lambda(\omega_\nu) &= \frac{1}{\beta}\sum_{\substack{\boldsymbol{k}\boldsymbol{p}\\ \rho,\omega_n}}|\tilde{T}^{\lambda\rho}_{\boldsymbol{k}\boldsymbol{p}}|^2\mathcal{G}^{\rm A}_\lambda(\boldsymbol{k},\omega_n-\omega_\nu)\mathcal{G}^{\rm B}_\rho(\boldsymbol{p},\omega_n),\\ \Psi_\lambda(\omega_\nu) &= \frac{1}{\beta}\sum_{\substack{\boldsymbol{k}\boldsymbol{p}\\ \rho,\omega_n}}\tilde{T}^{\lambda\rho}_{\boldsymbol{k}\boldsymbol{p}}\tilde{T}^{\lambda\rho}_{-\boldsymbol{k},-\boldsymbol{p}}\mathcal{F}^{\rm A*}_\lambda(\boldsymbol{k},\omega_n-\omega_\nu)\mathcal{F}^{\rm B}_\rho(\boldsymbol{p},\omega_n),\end{aligned} \tag{5}$$

where $\omega_\nu \to eV + \mathrm{i}\,0^+$ is a boson Matsubara frequency. We have assumed that the bulk Green's functions may be used, neglecting boundary effects. Such effects could however be of importance in these systems,[16] due to the possibility of subgap surface bound states or distortions of the order parameters close to the surface.

Time-reversal symmetry of $H_{\rm T}$ gives $\tilde{T}^{\lambda\rho}_{-\boldsymbol{k},-\boldsymbol{p}} = \tilde{T}^{\lambda\rho\,*}_{\boldsymbol{k}\boldsymbol{p}}t^{\rm A}_{\lambda,\boldsymbol{k}}t^{\rm B*}_{\rho,\boldsymbol{p}}$.[5] These phase factors will cancel the ones from the anomalous Green's functions in Equation (5), which shows that each band λ may behave as a singlet superconductor with gap function $\chi_{\lambda,\boldsymbol{k}}$.[2,5]

We take the continuum limit[31] and assume that $\mathcal{N}_\lambda(\xi,\Omega)$, the angle-resolved density of states in band λ in the non-superconducting phase, is constant.

The gap $\chi_\lambda(\xi,\Omega)$ depends on both energy and the direction of momentum. Neglecting the energy dependence

is standard.[25,26] The tunnelling matrix elements ensures that momenta approximately perpendicular to the interface will dominate.[22,23,24] We therefore let $\chi_\lambda(\xi,\Omega) \approx \chi_\lambda$, the value at the Fermi level and directions normal to the interface (remember that $\chi_{\lambda,\boldsymbol{k}} = \chi_{\lambda,-\boldsymbol{k}}$). This is exact if the gaps are isotropic or if the tunnelling is strictly one-dimensional. It could also be a good approximation if the variations of χ_+ and χ_- are small in the region around normal incidence. We define $\chi_\lambda = |\chi_\lambda| e^{i\vartheta_\lambda}$.

The energy dependence of the tunnelling matrix elements may be neglected. We will need the quantity $\tau_{\lambda\rho} = \int d\Omega^A \int d\Omega^B |\tilde{T}^{\lambda\rho}(\Omega^A,\Omega^B)|^2$. Let us look at a specific example, where $|T_{\boldsymbol{kp}}|^2 \sim |T|^2 \hat{k}_\perp \hat{p}_\perp \Theta(\hat{k}_\perp \hat{p}_\perp)$.[32] In addition, we choose the Rashba interaction $\boldsymbol{B}^A_{\boldsymbol{k}} = \alpha(\hat{\boldsymbol{n}}^A \times \boldsymbol{k})$.[19] We let $\hat{\boldsymbol{n}}^A$ and $\hat{\boldsymbol{n}}^B$, and consequently the nodes of $\boldsymbol{B}^A_{\boldsymbol{k}}$ and $\boldsymbol{B}^B_{\boldsymbol{p}}$, point parallel to the interface. The Rashba interaction appears to be an appropriate choice for $CePt_3Si$[4] and $Cd_2Re_2O_7$.[5] Define the angle ζ by $\cos\zeta = \hat{\boldsymbol{n}}^A \cdot \hat{\boldsymbol{n}}^B$. This gives

$$\tau_{\lambda\rho} = \frac{|T|^2}{2}\left(1 + x\,\lambda\rho\,\cos\zeta\right), \tag{6}$$

with $x \approx 0.6$.[33] Numerical integration indicates that Equation (6) is a very good approximation also when parallel momentum is conserved.[34] In general, it seems reasonable that if mostly electrons near normal incidence contribute to the current, Equation (6) is applicable with $\cos\zeta \equiv \hat{\boldsymbol{B}}^A_{\boldsymbol{q}_\perp} \cdot \hat{\boldsymbol{B}}^B_{\boldsymbol{q}_\perp}$ where $\boldsymbol{q}_\perp$ is perpendicular to the interface. $x \in [0,1]$ is in fact an experimentally accessible quantity. This will be discussed below.

The conductance in the normal phase is given by $G_N \equiv I_N/V = 2e^2\pi \sum_{\lambda\rho} \mathcal{N}^A_\lambda \mathcal{N}^B_\rho \tau_{\lambda\rho}$. Define $r_N(\zeta) \equiv G_N(\zeta)/G_N(\pi/2)$. Using Equation (6), we find $r_N(\zeta) = 1 + (1-d)^2/(1+d)^2\, x\cos\zeta$ where $d \equiv \mathcal{N}_-/\mathcal{N}_+$ is the ratio of the densities of states. The dependence on the angle ζ is similar to tunnelling magnetoresistance between ferromagnets[18] and vanishes if $\mathcal{N}_+ = \mathcal{N}_-$.

In the superconducting phase, the quasiparticle current for $T \to 0$ becomes $I_{qp} = \pi e \sum_{\lambda\rho} \mathcal{N}^A_\lambda \mathcal{N}^B_\rho \tau_{\lambda\rho} \gamma^{AB}_{\lambda\rho}$, where

$$\gamma^{AB}_{\lambda\rho} = \Theta\left(|eV| - (|\chi^A_\lambda| + |\chi^B_\rho|)\right) eV \sqrt{1-\delta^2_{\lambda\rho}} \tag{7}$$
$$\times \left[2E\left(\frac{1-\sigma^2_{\lambda\rho}}{1-\delta^2_{\lambda\rho}}\right) - \frac{\sigma^2_{\lambda\rho}-\delta^2_{\lambda\rho}}{1-\delta^2_{\lambda\rho}} K\left(\frac{1-\sigma^2_{\lambda\rho}}{1-\delta^2_{\lambda\rho}}\right)\right].$$

$K(m)$ and $E(m)$ are the complete elliptic integrals of the first and second kind, respectively.[27] We have defined $\sigma_{\lambda\rho} = (|\chi^A_\lambda| + |\chi^B_\rho|)/eV$ and $\delta_{\lambda\rho} = (|\chi^A_\lambda| - |\chi^B_\rho|)/eV$. This is a two-band generalisation of the one-band s-wave expression.[26] *The usual one-band treshold at* $eV = \chi^A + \chi^B$ *is replaced by at most four discontinuities.*

At zero voltage difference, the Josephson current becomes $I_J = 4\pi e \sum_{\lambda\rho} \mathcal{N}^A_\lambda \mathcal{N}^B_\rho \tau_{\lambda\rho} \Gamma^{AB}_{\lambda\rho} \sin(\vartheta^B_\rho - \vartheta^A_\lambda)$, where

$$\Gamma^{AB}_{\lambda\rho} = \frac{|\chi^A_\lambda||\chi^B_\rho|}{|\chi^A_\lambda| + |\chi^B_\rho|} K\left(\frac{(|\chi^A_\lambda| - |\chi^B_\rho|)^2}{(|\chi^A_\lambda| + |\chi^B_\rho|)^2}\right). \tag{8}$$

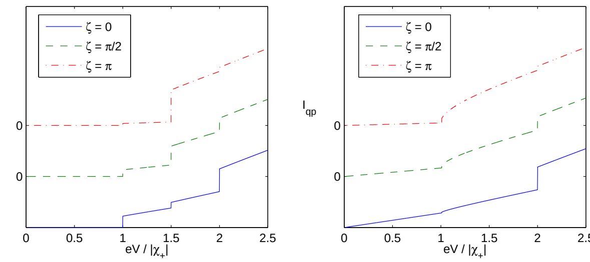

FIG. 1: (Color online) The quasiparticle current-voltage diagram at $T = 0$ for angles $\zeta = 0, \pi/2, \pi$, $x = 0.7$ and $d = 1$. The graphs are displaced in the vertical direction for clarity. *Left panel*: $|\chi_-|/|\chi_+| = 0.5$. *Right panel*: $|\chi_-|/|\chi_+| = 0.01$.

This is a general two-band s-wave expression valid when interband Cooper pairs are absent. It is a straightforward generalisation of the standard one-band result.[25,26]

We now consider the case of spin-orbit split bands. Usually, for equal systems one could let $|\chi^A_\pm| = |\chi^B_\pm| \equiv |\chi_\pm|$. This might not always be justified in our model, since the direction dependence of the gaps might depend on the nature of $\boldsymbol{B}^{A(B)}_{\boldsymbol{k}(\boldsymbol{p})}$. However, let us again turn to the example above, where $\hat{\boldsymbol{n}}^A$ and $\hat{\boldsymbol{n}}^B$ point parallel to the interface. This makes $\boldsymbol{k}, \boldsymbol{p} \sim \boldsymbol{q}_\perp$ equivalent directions even though $\hat{\boldsymbol{n}}^A \neq \hat{\boldsymbol{n}}^B$, at least in the isotropic approximation. The above assumption should then be justified and will be used below. In addition, since the model (2) contains interband pair scattering, we consider phase-locked bands where $\vartheta_+ = \vartheta_- + n\pi$ and n is zero or one. We do not investigate the possibility of small oscillations of the interband phase difference.[28]

Whenever $|\chi_+| = |\chi_-|$ and $\mathcal{N}_+ = \mathcal{N}_-$, I_{qp} becomes independent of ζ and equals the one-band result. In the case of unequal gaps, extra discontinuities should appear in the current-voltage characteristics. Figure 1 shows the quasiparticle current at $T = 0$ as function of voltage for the three angles $\zeta = 0$, $\pi/2$ and π with $x = 0.7$ and $d = 1$. The graphs are displaced in the vertical direction for clarity. Note that as $x \to 0$ all cases approach the $\zeta = \pi/2$ graph. In the left panel, $|\chi_-|/|\chi_+| = 0.5$. Discontinuities appear at $eV = 2|\chi_-|$, $(|\chi_+| + |\chi_-|)$ and $2|\chi_+|$. In the right panel, where $|\chi_-|/|\chi_+| = 0.01$, current can flow also for small voltages, although this almost vanishes as $\zeta \to \pi$. Smearing of the steps due to interband scattering of quasiparticles should be negligible as long as the gap difference is well above $k_B T$. This breaks down in the limit of equal gaps, but then the current-voltage diagram collapse to the one-band result anyway. Anisotropic gaps, gap nodes and non-zero temperature may in general also lead to smearing. At non-zero temperatures, logarithmic singularities in I_{qp} at $eV = ||\chi_+| - |\chi_-||$ may show up.[29]

Define $|\chi_M| = \max(|\chi_+|, |\chi_-|)$ and $|\chi_m| = \min(|\chi_+|, |\chi_-|)$. The ratio $F \equiv |\chi_m|/|\chi_M|$ is easily found from the position of the first and last discontinuity. In addition, if $D_{\lambda\rho}$ denotes the jump in I_{qp} at $eV = (|\chi_\lambda| + |\chi_\rho|)$, one finds $d^2 F = D_{mm}/D_{MM}$, where $d \equiv \mathcal{N}_m/\mathcal{N}_M$.

These methods for finding F and d^2F are independent of ζ and x. Furthermore, x may be determined from $x\cos\zeta = (2\sqrt{D_{\rm mm}D_{\rm MM}} - D_{+-})/(2\sqrt{D_{\rm mm}D_{\rm MM}} + D_{+-})$, for any angle ζ.

The critical Josephson current $I_{\rm J,c}$ will also depend on ζ. There is a close analogy to tunnelling magnetoresistance.[18] Define $a = (-1)^n 8\mathcal{N}_+\mathcal{N}_-\Gamma_{+-}/\pi(\mathcal{N}_+^2|\chi_+| + \mathcal{N}_-^2|\chi_-|)$, where $|a| = |a|(F,d)$ is monotonically increasing for $F \leq 1$, $|a|(0,d) = 0$ and $|a|(1,d) = 2d/(1+d^2) \leq 1$. We find that $a = -1$ results in $I_{\rm J,c}(\pi/2) = 0$. For $a \neq -1$, we define

$$r_{\rm J}(\zeta) \equiv \frac{I_{\rm J,c}(\zeta)}{I_{\rm J,c}\left(\frac{\pi}{2}\right)} = \left|1 + \frac{1-a}{1+a}\, x\cos\zeta\right|, \qquad (9)$$

showing the possible modulation of the critical Josephson current with ζ. In addition, $a = (1 - r_{\rm J}(0) + x)/(r_{\rm J}(0) - 1 + x)$. The sign of a determines n and hence the relative sign between $\chi_{+,\boldsymbol{q}_\perp}$ and $\chi_{-,\boldsymbol{q}_\perp}$. *One may then determine the ratio between the singlet and triplet components of the order parameters in a spin basis, since* $|\eta_{\boldsymbol{q}_\perp,\rm S}|/|\eta_{\boldsymbol{q}_\perp,\rm T}| = (1 + (-1)^n F)/(1 - (-1)^n F)$.

Note that if $|\chi_+| = |\chi_-|$ and $n = 0$, $r_{\rm J}(\zeta) = r_{\rm N}(\zeta)$. If in addition $d = 1$, $I_{\rm J,c}$ becomes independent of ζ and the one-band result[25] is recovered. A modulation may be a result of unequal gaps ($F \neq 1$), unequal densities of states ($d \neq 1$) or both. In addition, no modulation of $r_{\rm J}(\zeta)$ could be interpreted both as $x = 0$ and $a = 1$. Both these ambiguities should be distinguishable through the quasiparticle current-voltage characteristics. Consistency demands that $|a|(F,d)$ found from the Josephson current fits F and d found from the quasiparticle current.

To determine that the jumps in the quasiparticle current arise from spin-orbit split bands due to breakdown of inversion symmetry along a certain axis, several junctions with different relative orientations of those axes would be needed.[35] The synthesis and manipulation of such junctions thus represents a considerable experimental challenge. However, building Josephson junctions with controllable crystallographic orientations was essential to proving the d-wave symmetry of the order parameter in the high-T_c cuprates.[30] Also, the presence of two gaps may be possible to detect in other experiments, such as scanning tunnelling microscopy.

In conclusion, we predict possible new effects in the tunnelling current between non-centrosymmetric superconductors with significant spin-orbit splitting. Spin conservation in the tunnelling barrier may then result in a modulation of the critical Josephson current when varying the relative angle between the vectors describing absence of inversion symmetry on each side. We have also shown that several discontinuities may appear in the quasiparticle current. We have argued that both these phenomena might help to determine the possibly exotic gap symmetry and pairing state of non-centrosymmetric superconductors.

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[31] $\sum_{\boldsymbol{k}} f_\lambda(\boldsymbol{k}) \rightarrow \int \mathrm{d}\xi \int_{S(\xi)} \mathrm{d}\Omega \, \mathcal{N}_\lambda(\xi,\Omega) \, f_\lambda\left(\boldsymbol{k}(\xi,\Omega)\right)$. $S(\xi)$ is a constant energy surface in momentum space. $\mathcal{N}_\lambda(\xi,\Omega)$ is the density of states.
[32] The Heaviside function Θ ensures that the momentum perpendicular to the interface does not change sign.[22]
[33] We find $x = (2\int_0^1 \mathrm{d}y\sqrt{1-y}K(y)/\pi)^2 \approx 0.6$, where $K(y)$ is the complete elliptic integral of the first kind.[27]
[34] $|T_{\boldsymbol{kp}}|^2 \sim |T|^2 \hat{k}_\perp \delta_{\boldsymbol{k},\boldsymbol{p}}$ gives the same qualitative result. Refraction effects are not included, but this is negligible whenever $(|\boldsymbol{k}_{\mathrm{F},-}| + |\boldsymbol{k}_{\mathrm{F},+}|)/2 \gg ||\boldsymbol{k}_{\mathrm{F},-}| - |\boldsymbol{k}_{\mathrm{F},+}|| \sim \alpha$, at least in the important region, *i.e.* close to normal incidence.
[35] In our model, this amounts to controlling the angle between $\hat{\boldsymbol{n}}^{\mathrm{A}}$ and $\hat{\boldsymbol{n}}^{\mathrm{B}}$.